



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2017

MATHEMATICS P1

MARKS: 150

TIME: 3 hours



This question paper consists of 9 pages including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.
3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
4. Answers only will not necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.
9. An information sheet with formulae is included at the end of the question paper.

QUESTION 1

Solve for x :

1.1 $2x(x + 1) - 7(x + 1) = 0$ (3)

1.2 $x^2 - 5x - 1 = 0$, correct to two decimal places. (3)

1.3 $4x^2 + 1 \geq 5x$ (4)

1.4 $5^{4x+3} \cdot 100^{-2x+1} = 50\,000$ (4)

1.5 Solve for x and y simultaneously.

$$x = 2y \quad \text{and} \quad x^2 + 2x - y - y^2 = 36 \quad (5)$$

1.6 Show that the roots of $x^2 - kx + k - 1 = 0$ are real and rational for all ~~real~~ ^{rational} values of k . (4)

[23]

QUESTION 2

2.1 Given the following linear series: $3 + 7 + 11 + \dots + 483$

2.1.1 How many terms does the above series have? (3)

2.1.2 Write the above series in sigma notation. (2)

2.2 $2t - 4$; $t - 3$; $8 - 2t$ are the 10th, 11th and 12th terms of an arithmetic sequence.

2.2.1 Determine the value of t . (3)

2.2.2 Calculate the value of the first term. (3)

2.3 The following information of a geometric pattern is given:

$$T_1 + T_2 = -1 \quad \text{and} \quad T_3 + T_4 = -4$$

Determine the numerical values of the first three terms if $r > 0$. (6)

[17]

QUESTION 3

Given the following list of numbers in a quadratic sequence:

41 ; 43 ; 47 ; 53 ; 61 ; 71 ; 83 ; 97 ; 113 ; 131

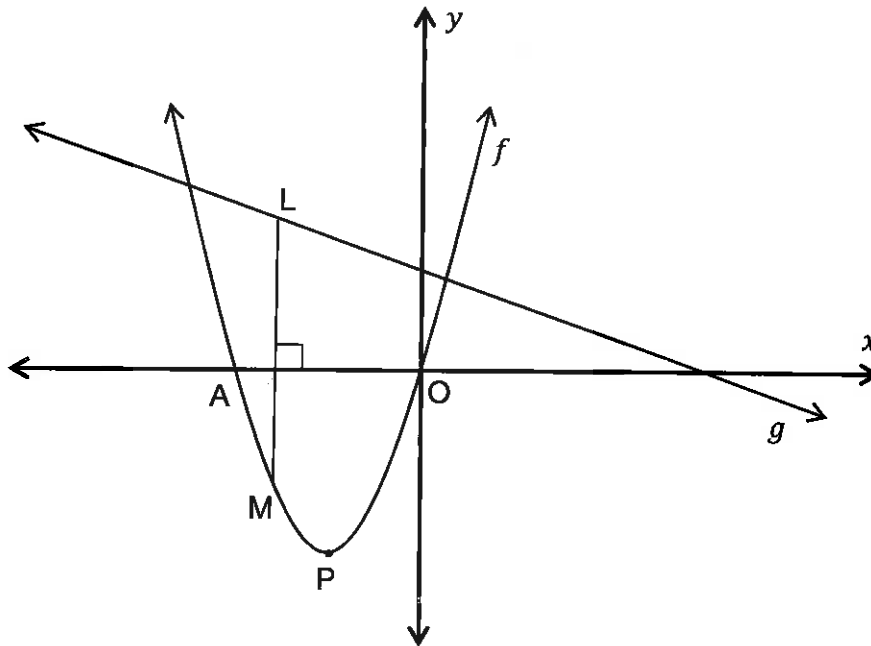
- 3.1 Determine the general term of the above sequence if it has a constant second difference. (5)
- 3.2 Calculate T_{41} and show that it is not a prime number. (3)
- 3.3 Determine the unit digit of the 49 999 998th term of the above sequence. (3)
- [11]

QUESTION 4

- 4.1 Wayde invest R500 000 at 7,2% per annum compounded monthly.
- 4.1.1 Write down an expression for the value of his investment after n full years. (2)
- 4.1.2 Determine the value of his investment after 5 full years. (2)
- 4.1.3 If the investment exceeds R1 million rand after n full years, calculate the value of n . (3)
- 4.2 Mr Jones wants to purchase a new car. The car cost R350 000 and he wants to pay R10 000 per month for three years. The interest rate on a financed loan, payable monthly, is 15% p.a. compounded monthly.
- 4.2.1 Calculate how much he will have to give as a deposit, to the auto dealer. (5)
- 4.2.2 Calculate his monthly payment if he pays no deposit and the car is financed over 5 years, at an interest rate of 18,5% p.a. compounded monthly. (4)
- [16]

QUESTION 5

The sketch shows the graph of $f(x) = x(x + 3)$ and $g(x) = -\frac{1}{2}x + 2$.



- 5.1 Determine the coordinates of A. (1)
- 5.2 Calculate the coordinates of P, the turning point of f . (3)
- 5.3 Determine the average gradient of f between $x = -5$ and $x = -3$. (3)
- 5.4 Determine the value(s) of x for which $f(x) > 0$. (2)
- 5.5 Determine the coordinates of the turning point of h if $h(x) = f(x - 2)$. (2)
- 5.6 L is a point on the straight line and M is a point on the parabola. LM is perpendicular to the x -axis. Show that the length LM can be written as:

$$LM = -\left(x + \frac{7}{4}\right)^2 + \frac{81}{16} \quad (4)$$

[15]

QUESTION 6

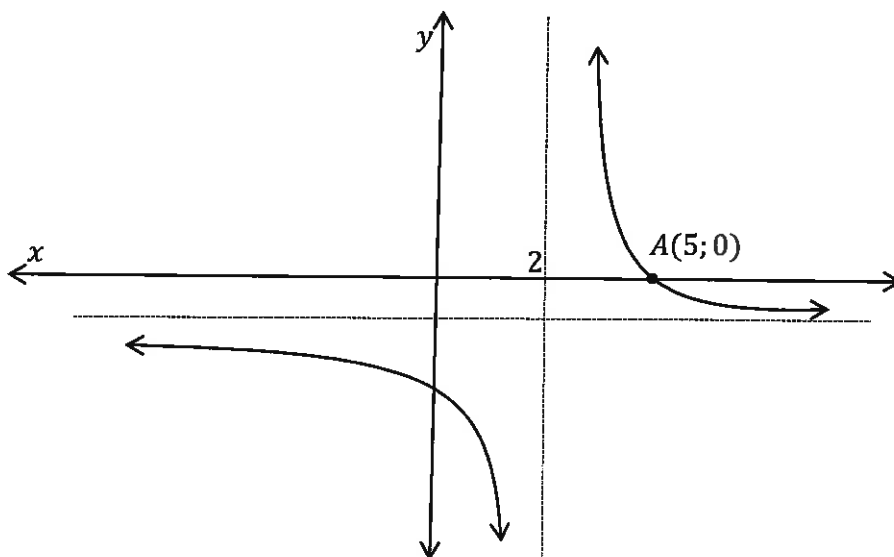
Given: $h(x) = 2^{-x}$

- 6.1 Draw a neat sketch of h . (3)
- 6.2 Determine the equation of q , the graph obtained by reflecting h in the line $y = 0$. (1)
- 6.3 Write down the equation of h^{-1} , the inverse of h , in the form $y = \dots$ (2)
- 6.4 Write down the range of h . (1)
- 6.5 Sketch the graph of h^{-1} on the same set of axes as h . (2)
- 6.6 Determine the x -value(s) for which $h^{-1}(x) \geq -3$. (2)
- [11]

QUESTION 7

A sketch of the hyperbola $f(x) = \frac{d-x}{x-p}$, where p and d are constants, is given below.

The dotted lines are the asymptotes. The point $A(5; 0)$ is given on the graph of f .



- 7.1 Determine the values of d and p . (2)
- 7.2 Show that the equation can be written as $y = \frac{3}{x-2} - 1$ (2)
- 7.3 Write down the image of A if A is reflected about the axis of symmetry $y = x - 3$. (2)
- [6]

QUESTION 8

8.1 Given: $f(x) = -2x^2 + p$

Determine $f'(x)$ from first principles. (5)

8.2 Determine: $D_x \left[4\sqrt[3]{x} + \frac{1}{3x} + 2 \right]$ (4)

[9]

QUESTION 9

Given: $f(x) = (x - 1)^2(x + 3)$

9.1 Determine the turning points of f . (5)

9.2 Draw a neat sketch of f showing all intercepts with the axes as well as the turning points. (4)

9.3 Determine the coordinates of the point where the concavity of f changes. (3)

9.4 Determine the value(s) of k , for which $f(x) = k$ has three distinct roots. (2)

9.5 Determine the equation of the tangent to f that is parallel to the line $y = -5x$ if $x < 0$. (6)
[20]

QUESTION 10

A closed rectangular box, with a rectangle as base, has a length $(2x)$ cm and width (x) cm. The total surface area (all 6 sides) is 243 cm^2 .

10.1 Show that the height, h , is equal to $\left(\frac{81}{2x} - \frac{2x}{3} \right) \text{ cm}$. (2)

10.2 Show that the volume of the box, in terms of x , is given by the formula:
 $V = 81x - \frac{4}{3}x^3$ (2)

10.3 Calculate the value of x if the volume of the box is a maximum. (3)
[7]

QUESTION 11

- 11.1 You have to select a new password for your “Dropbox” account on your computer. The password must consist of 3 digits and 2 letters of the alphabet in that order. The digit 0(zero) is not allowed and no consonants are allowed. Any digit may be repeated but the vowels may not be repeated. How many different passwords are possible? (3)
- 11.2 Using the letters in the word **FUNDAMENTALS** , determine:
- 11.2.1 The number of unique 12 letter arrangements that can be formed (3)
- 11.2.2 The probability that a new arrangement will start and end with the letter **N** (3)
- 11.3 In a city where the ratio of male to female is 1 : 2, one person is selected to flip a fair coin for the kick-off of a soccer tournament.
- 11.3.1 Draw a tree diagram to show all possible outcomes of the coin toss. (3)
- 11.3.2 Determine the probability that it will be a woman who flips the coin. (1)
- 11.3.3 Determine the probability that it will be a man who flips a head. (2)
- [15]**
- TOTAL: 150**

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$